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HCU-003-1013008 Seat No. ____________

B. Sc. (Sem. III) (CBCS) (W.I.F. - 2016) Examination

October/November - 2017

Mathematics : MATHS-03 (A)

(Real Analysis) (New Course)

Faculty Code : 003

Subject Code : 1013008

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction : All questions are compulsory.

1 (a) Answer the following questions briefly : **4**

(1) Define : Bounded above sequence.

(2) Define : Increasing sequence.

(3) Is the sequence $\left\{i^n\right\}$ convergent ? $\left(i = \sqrt{-1}\right)$.

(4) Find limit of the Sequence $\left\{1+\frac{(-1)^n}{n}\right\}$.

(b) Attempt any one out of two : **2**

(1) Discuss the convergence of the sequence $\left\{\sqrt{n+1}-\sqrt{n}\right\}$.

(2) Show that $\lim\limits_{n\to\infty}\frac{1}{n}\left[1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+......+\frac{1}{n}\right]=0$.

(c) Attempt any one out of two : **3**

(1) Discuss the convergence of the sequence $\{S_n\}$ where

$$S_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^2} + \ldots\ldots + \frac{1}{3^n}.$$

(2) Prove that every convergent sequence is bounded.

(d) Attempt any one out of two : **5**

(1) Discuss the convergence of the sequence $\{S_n\}$ where

$S_1 = 4, S_{n+1} = 3 - \frac{2}{S_n}$ and find its limit.

(2) Prove that every convergent sequence has a unique limit.

2 (a) Answer the following questions briefly : **4**

(1) Is the series $\sum_{n=1}^{\infty} \frac{n}{n^{3/2}}$ convergent ?

(2) Show that the series 1+2+3+4+.........is divergent.

(3) Is the series $\sum \frac{n^2+5}{3n^2+4}$ convergent ?

(4) Is the following statement is true ? If $\lim_{n\to\infty} a_n = 0$ then $\sum a_n$ is convergent.

(b) Attempt any one out of two : **2**

(1) Discuss the convergence of series $\sum \frac{1}{n^2} \sin \frac{1}{n}$.

(2) Discuss the convergence of series $\sum \frac{n^2}{3^n}$.

(c) Attempt any one out of two : **3**

(1) Discuss the convergence of series

$$\frac{1}{4.6}+\frac{\sqrt{3}}{6.8}+\frac{\sqrt{5}}{8.10}+\frac{\sqrt{7}}{10.11}+.......$$

(2) Discuss the convergence of series

$$\frac{1}{1+2}+\frac{2}{1+2^2}+\frac{3}{1+2^3}+\frac{4}{1+2^4}+......$$

(d) Attempt any one out of two : **5**

(1) Find radius of convergence and interval of convergence of the series $\sum \frac{(-3)^n x^n}{\sqrt{n+1}}$.

(2) Show that the series $\sum(-1)^n\left(\sqrt{n^2+1}-n\right)$ is conditionally convergent.

3 (a) Answer the following questions briefly : **4**

(1) Define : Divergence.

(2) Define : Solenoid function.

(3) If $\phi(x, y, z)=x^3+y^3+z^3$ then find $grad\phi$.

(4) Define : Laplace equation.

(b) Attempt any one out of two : **2**

(1) If $\overline{f}=\left(x^2 y, -2xz, 2yz\right)$ then find $curl\overline{f}$.

(2) If ϕ and ψ are differential scalar function on the domain D of R^3 then prove that

$$\nabla\left(\frac{\phi}{\psi}\right)=\frac{\psi\nabla\phi-\phi\nabla\psi}{\psi^2}.$$

(c) Attempt any one out of two : **3**

(1) If $u = \log\left(x^2 + y^2 + z^2\right)$ then find $div(gradu)$ at the point (1, 2, 3).

(2) In usual notation, prove that $div\left(\frac{\overline{r}}{r^3}\right) = 0$.

(d) Attempt any one out of two : **5**

(1) If $\overline{f}$ and $\overline{g}$ are vector function on the domain D whose first order partial derivatives exists then prove that

$$div\left(\overline{f} \times \overline{g}\right) = \overline{g}.curl\overline{f} - \overline{f} \cdot curl\,\overline{g}$$

(2) In usual notation, prove that $div\left(r^n \overline{r}\right) = (n+3)\,r^n$.

4 (a) Answer the following questions briefly : **4**

(1) Evaluate $\int_0^1 \int_0^1 \int_0^1 xydxdydz$.

(2) Write $|J|$ by change of variable from Cartesian to cylindrical coordinate.

(3) Evaluate $\int_0^2 \int_0^1 \left(x^2 + 2y\right) dxdy$.

(4) Evaluate $\int_0^1 \int_0^x \left(x^2 + y^2\right) dydx$.

(b) Attempt any one out of two : **2**

(1) Evaluate $\int_0^1 \int_0^1 \int_0^1 (x+y+z)\,dxdydz$.

(2) Prove that $\int_0^1 \int_0^{\sqrt{1-x^2}} x^2 dydx = \frac{\pi}{16}$.

(c) Attempt any one out of two : **3**

(1) Evaluate $\int_0^a \int_{\sqrt{y}}^3 \sin\left(\pi x^3\right) dxdy$ by changing the order of integral.

(2) Prove that $\iint_R (y-x)\,dxdy = -2$, where R is bounded by lines $y = x-3,\ y = x+2,\ 3y+x = 5,\ 3y+x = 7$.

(d) Attempt any one out of two : **5**

(1) Prove that $\iint_R (x-y)^2 \sin^2(x+y)\,dxdy = \frac{\pi^4}{3}$ where R is parallelogram whose vertices are $(\pi, 0), (2\pi, \pi), (\pi, 2\pi)\,(0, \pi)$.

(2) Prove that the volume of ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is $\frac{4}{3}\pi abc$.

5 (a) Answer the following questions briefly : **4**

(1) $\overline{\left|\frac{1}{4}\right.}\overline{\left|\frac{3}{4}\right.} =$ ________.

(2) State relation between Beta and Gamma Function.

(3) State Stoke's Theorem.

(4) State Green's Theorem.

(b) Attempt any one out of two : **2**

(1) Prove that $\int_0^\infty e^{-ax} x^{n-1} dx = \frac{\overline{|n}}{a^n}$, where $a > 0, n > 0$.

(2) Find $\int_{(2,1)}^{(1,2)} y dx$.

(c) Attempt any one out of two : **3**

(1) In usual notation prove that $\overline{|p+1} = p!$, where $p > 0$.

(2) Prove that the line integral

$\int_{(0,0)}^{(x,y)} \left(6xy^2 - y^3\right) dx + \left(6x^2 y - 3xy^2\right) dy$ is independent of path and also find its value.

(d) Attempt any one out of two : **5**

(1) (i) Find $\iint_S x^2 dydz + y^2 dzdx + 2z(xy - x - y)\, dxdy$, where $S : 0 \le x, y, z \le 1$ is a solid surface.

(ii) Find $\int_C x^2 y^3 dx + dy + zdz$, where $C : x^2 + y^2 = a^2, z = 0$ is a circle.

(2) Prove that $\beta(p, q) = \int_0^1 \frac{x^{p-1} + x^{q-1}}{(1+x)^{p+q}} dx$, where $p > 0$ and $q > 0$.